Homework exercises for lectures #10–13

1. (cf. Milne Exercise III.1.31 a–c, pp. 94–95) Let A be a noetherian ring and let it also denote the corresponding constant sheaf on $X_{\text{ét}}$. If F and G are sheaves of A-modules on $X_{\text{ét}}$, show that

$$\underline{\mathrm{Ext}}^p_A(F,G)_{\overline{x}} = \mathrm{Ext}^p_A(F_{\overline{x}}, G_{\overline{x}})$$

if F is pseudocoherent. Also show that

$$\underline{\mathrm{Ext}}^p_A(F,G) = 0,$$

for all $p > 0$ if either

(i) F is a locally free sheaf of A-modules of finite rank, or

(ii) F is pseudocoherent and G is the constant sheaf of an injective A-module.

2. (cf. Milne Exercise III.2.24, pp. 110) Show that writing $X = U_0 \cup U_1$ as a union of Zariski-open subsets, produces a long exact sequence for sheaves on $X_{\text{ét}}$: the Mayer–Vietoris sequence. Milne suggests a proof using the spectral sequence from Čech cohomology to usual cohomology.

Aside: if we let $j_i\colon U_i \to X$ denote the inclusions, is it true that the sequence

$$0 \to F \to j_{0*}j_0^*F \oplus j_{1*}j_1^*F \to j_{01*}j_{01}^*F \to 0$$

is exact? Can this be used to deduce the result?

3. Read about twisted forms (Milne p. 134), and do Milne's Exercise III.4.24 a–c on Brauer–Severi schemes and their duals.